Topic → Problem based on linear Differential Equation with constant co-efficient.

if the Equation is of the form

$f(D)y = \sin ax$ provided $f(-a^2) \neq 0$

or $f(D)y = \cos ax$

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By - Samrendra kumar

Solution → Method for

C.F $\Rightarrow f(D) = 0$

for P.I $= \frac{1}{f(D)} \sin ax$ or $\frac{1}{f(D)} \cos ax$

$= \frac{1}{g(D^2)} \sin ax$ or $\frac{1}{g(D^2)} \cos ax$

$f(D) = g(D^2)$

$= \frac{1}{g(-a^2)} \sin ax$ or $\frac{1}{g(-a^2)} \cos ax$

provided $g(-a^2) \neq 0$.

Problem 1 — Solve.

$$\frac{d^2y}{dx^2} + y = \sin 2x$$

Solution → the given Equation is in symbolic form

$(D^2+1)y = \sin 2x$

Here

$f(D) = D^2+1$

Now for C.F

$f(D) = 0$

$D^2+1 = 0$

$D = \pm i$

$\therefore$ C.F $= C_1\cos x + C_2 \sin x$

for P.I –

$\frac{1}{f(D)} \sin 2x$

$f(-2^2) = f(-4) = -$

$= -$

$\frac{1}{D^2+1} \sin 2x$

$= \frac{1}{f(-2^2)+1} \sin 2x$

$= \frac{1}{-3} \sin 2x$

$\therefore$ Complete Solution

$= C.F + P.I$

$= C_1\cos x + C_2\sin x - \frac{1}{3}\sin 2x$

2.

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 5y = 10\sin x$$

The Given equation can be written in symbolic form as

$$(D^2 - 2D + 5)y = 10\sin x$$

$$f(D) = D^2 - 2D + 5$$

$$f(-1^2) = -1 - 2D + 5$$

$$= 4 - 2D$$

for P.I $= \frac{1}{4-2D} 10\sin x$

$$= \frac{4+2D}{16-4D^2} 10\sin x \qquad D^2 = -1$$

$$= \frac{(4+2D)\,10\sin x}{\cancel{20}\,2}$$

$$= \frac{1}{2}[4+2D]\sin x$$

$$= \frac{1}{2}\left[4\sin x + 2\frac{d}{dx}\sin x\right]$$

$$= \frac{1}{2}[4\sin x + 2\cos x]$$

$$= 2\sin x + \cos x$$

for C.F $\quad f(D) = 0 \Rightarrow D^2 - 2D + 5 = 0$

$\therefore CF = e^x(c_1\cos 2x + c_2\sin 2x) \qquad D = \frac{2 \pm \sqrt{4-20}}{2} = \frac{2 \pm 4i}{2} = 1 \pm 2i$

Ans

Complete Solution

$y = C.F + P.I$

$= e^{x}(C_1 \cos 2x + C_2 \sin 2x) + 2 \sin x + \cos x$

Q. Solve

$\frac{d^2y}{dx^2} + y = \cos 2x$

Ans Given Equation is written in Symbolic form as

$\cancel{\frac{d^2y}{dx^2}} = (D^2+1)y = \cos 2x$

$f(D) = D^2+1$ $\qquad f(D^2) = -4+1 = -3$

$f(-2^2) = -4+1 = -3$

$\therefore P.I = \frac{1}{f(-2^2)} \cos 2x$

$= -\frac{1}{3} \cos 2x$

for C.F $\qquad f(D) = 0$

$D^2+1 = 0$

$D = \pm i$

$\therefore C.F = C_1 \cos x + C_2 \sin x$

$\therefore$ Complete Solution $= C.F + P.I$

$= C_1 \cos x + C_2 \sin x - \frac{1}{3} \cos 2x$